Second Iterim Report (Item No. 0001AB) on

"An Efficient Numerical Algorithm for Solving Scattering and Inverse Scattering Problems of Electromagnetic Waves"

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The scattering of normal incident time-harmonic TEM electromagnetic wave by a cylindrical target with axis along z-direction is considered, e.g., $\underline{E} = E_x\underline{i}_x + E_y\underline{i}_y$ and $\underline{H} = H_z\underline{i}_z$, where $\underline{i}_a$ is the unit vector in the $\underline{a}$-direction. The whole space domain Ω is divided into three connected but non-overlapping sub-domains, the interior region Ω_1 representing the target and possessing a non-orthogonal cylindrical grid system centered in itself, the intermediate region Ω_2 representing the free space just outside of the target and possessing the same grid system, and the exterior region Ω_3 representing the far-field free space but truncated at a large distance away from the target and possessing the standard orthogonal cylindrical grid system (Fig. 1).

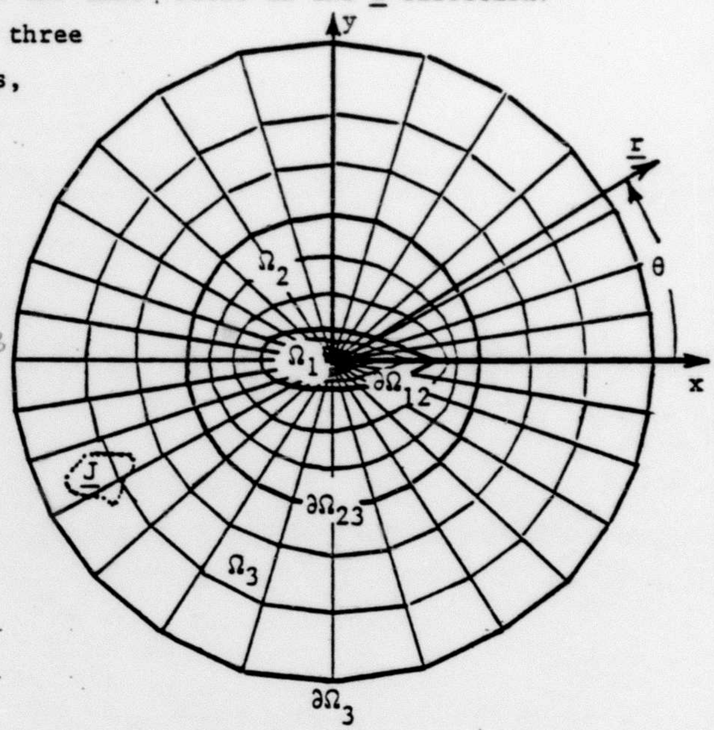


Fig. 1

To facilitate the discretization of the Maxwell's equations on the non-orthogonal grid system, the following integral forms of the Maxwell's equations are used.

$$\begin{cases} \oint \underline{E}_1 \cdot d\underline{\ell} = i\omega \oiint \underline{\underline{\mu}}\,\underline{H}_1 \cdot d\underline{s}, \\ \oint \underline{H}_1 \cdot d\underline{\ell} = \oiint (\sigma - i\omega\underline{\underline{\varepsilon}}) \cdot \underline{E}_1 \cdot d\underline{s}, \end{cases} \quad \underline{x} \in \Omega_1,$$
$$\begin{cases} \oint \underline{E}_2 \cdot d\underline{\ell} = i\omega \oiint \mu_o \underline{H}_2 \cdot d\underline{s}, \\ \oint \underline{H}_2 \cdot d\underline{\ell} = -i\omega\varepsilon_o \oiint \underline{E}_2 \cdot d\underline{s}, \end{cases} \quad \underline{x} \in \Omega_2, \qquad (1)$$
$$\begin{cases} \oint \underline{E}_3 \cdot d\underline{\ell} = i\omega\mu_o \oiint \underline{H}_3 \cdot d\underline{s}, \\ \oint \underline{H}_3 \cdot d\underline{\ell} = \oiint (\underline{J} - i\omega\varepsilon_o \underline{E}_3) \cdot d\underline{s}, \end{cases} \quad \underline{x} \in \Omega_3,$$

with boundary conditions at $\partial\Omega_{12}$,

$$\underline{n} \times \underline{E}_2 = \underline{n} \times \underline{E}_1, \qquad \underline{n} \times \underline{H}_2 = \underline{n} \times \underline{H}_1,$$
$$\varepsilon_o \underline{E}_2 \cdot \underline{n} = \underline{\underline{\varepsilon}} \underline{E}_1 \cdot \underline{n}, \qquad \mu_o \underline{H}_2 \cdot \underline{n} = \underline{\underline{\mu}} \underline{H}_1 \cdot \underline{n}, \tag{2}$$

and the asymptotic terminating condition at $\partial\Omega_3$,

$$\underline{n} \times \underline{E}_3 = (\mu_o/\varepsilon_o)^{\frac{1}{2}}(\underline{n} \times \underline{H}_3), \tag{3}$$

where $\underline{n}$ is the unit normal vector at the interfaces and

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \qquad \underline{\underline{\mu}} = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix}.$$

Eq. (1) is discretized by using the rectangle rule on the line integral around the edges of all incremental quadrilateral defined by the grid system. Let each grid point of the non-orthogonal polar grid system be denoted by $(r_{ij}, \theta_j) \equiv (i,j)$, where "i" and "j" denote the i-th closed cylindrical grid line and the j-th radial grid line respectively; let the center of the quadrilateral defined by (i,j), $(i+1,j)$, $(i,j+1)$ and $(i+1,j+1)$ be denoted by $(i+\frac{1}{2}, j+\frac{1}{2})$; let $\Delta\ell_{\alpha,\beta}$ be the incremental distance between the points α and β, and ΔA_γ be the area of the incremental quadrilateral centered at γ. Moreover, let $i = 1,2,3,\ldots,I$, and $j = 1,2,3,\ldots,J$.

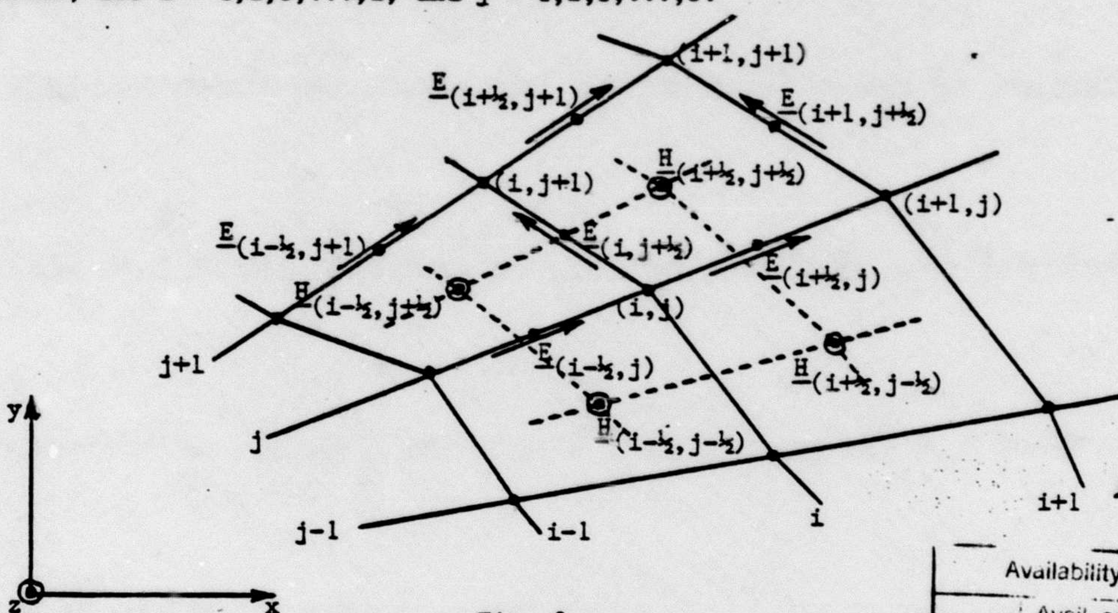


Fig. 2

In the neighborhood of (i,j) of Ω_1, the typical discretized (1) is

$$E_{i+\frac12,j}(r_{i+1,j}-r_{i,j}) - E_{i+\frac12,j+1}(r_{i+1,j+1}-r_{i,j+1})$$
$$+ E_{i+1,j+\frac12}\Delta\ell_{i+1,j+\frac12} - E_{i,j+\frac12}\Delta\ell_{i,j+\frac12} = i\omega\mu_{z,i+\frac12,j+\frac12}\, H_{i+\frac12,j+\frac12}\Delta A_{i+\frac12,j+\frac12},$$

$$(H_{i-\frac12,j+\frac12} - H_{i+\frac12,j+\frac12})\frac{(r_{i,j+1}+r_{i,j})4\sin\frac12(\theta_{j+1}-\theta_j)}{r_{i+1,j+1}+r_{i+1,j}-r_{i-1,j+1}-r_{i-1,j}}$$
$$= \sigma_{i,j+\frac12}\{\Delta\ell_{i,j+\frac12}E_{i,j+\frac12} - \tfrac14(E_{i-\frac12,j}+E_{i+\frac12,j}+E_{i-\frac12,j+1}+E_{i+\frac12,j+1})(r_{i,j+1}-r_{i,j})\cos\tfrac12(\theta_{j+1}-\theta_j)\}$$
$$- i\omega\{\Delta\ell_{i,j+\frac12}E_{i,j+\frac12}(\varepsilon_{x,i,j+\frac12}\sin^2\theta_{j+\frac12} + \varepsilon_{y,i,j+\frac12}\cos^2\theta_{j+\frac12})$$
$$- \tfrac14(E_{i-\frac12,j}+E_{i+\frac12,j}+E_{i-\frac12,j+1}+E_{i+\frac12,j+1})$$
$$\cdot(\varepsilon_{x,i,j+\frac12}(r_{i,j+1}\sin\theta_{j+1}-r_{i,j}\sin\theta_j)\sin\theta_{j+\frac12}$$
$$+ \varepsilon_{y,i,j+\frac12}(r_{i,j+1}\cos\theta_{j+1}-r_{i,j}\cos\theta_j)\cos\theta_{j+\frac12})\}\ .$$

in the neighborhood of (i,j) of Ω_2, the typical discretized (1) is

$$E_{i+\frac12,j}(r_{i+1,j}-r_{i,j}) - E_{i+\frac12,j+1}(r_{i+1,j+1}-r_{i,j+1})$$
$$+ E_{i+1,j+\frac12}\Delta\ell_{i+1,j+\frac12} - E_{i,j+\frac12}\Delta\ell_{i,j+\frac12} = i\omega\mu_o H_{i+\frac12,j+\frac12}\Delta A_{i+\frac12,j+\frac12},$$

$$(H_{i-\frac12,j+\frac12} - H_{i+\frac12,j+\frac12})\ \frac{4(r_{i,j+1}+r_{i,j})\sin\frac12(\theta_{j+1}-\theta_j)}{r_{i+1,j+1}+r_{i+1,j}-r_{i-1,j+1}-r_{i-1,j}}$$
$$= -i\omega\varepsilon_o\{\Delta\ell_{i,j+\frac12}E_{i,j+\frac12} - \tfrac14(r_{i,j+1}-r_{i,j})\ \cos\tfrac12(\theta_{j+1}-\theta_j)$$
$$\cdot(E_{i-\frac12,j}+E_{i+\frac12,j}+E_{i-\frac12,j+1}+E_{i+\frac12,j+1})\};$$

and in the neighborhood of (i,j) of Ω_3, the typical discretized (1) is

$$E_{i+\frac12,j}(r_{i+1}-r_i) - E_{i+\frac12,j+1}(r_{i+1}-r_i) + E_{i+1,j+\frac12}(\theta_{j+1}-\theta_j)r_{i+1}$$
$$- E_{i,j+\frac12}(\theta_{j+1}-\theta_j)r_i = i\omega\mu_o\tfrac12(\theta_{j+1}-\theta_j)(r_{i+1}-r_i)(r_{i+1}-r_i)H_{i+\frac12,j+\frac12},$$

$$(H_{i-\frac12,j+\frac12} - H_{i+\frac12,j+\frac12})\ \frac{4r_i\ \sin\frac12(\theta_{j+1}-\theta_j)}{r_{i+1}-r_{i-1}}$$
$$= -\ i\omega\varepsilon_o\ r_i(\theta_{j+1}-\theta_j)E_{i,j+\frac12} + \text{source terms due to the presence of } \underline{J},$$

where $\Delta\ell_{i,j+\frac12} = \frac12(r_{i,j+\frac12}+r_{i,j})(\theta_{j+1}-\theta_j)$

and $\Delta A_{i+\frac12,j+\frac12} = \frac14(\theta_{j+1}-\theta_j)\{(r_{i+1,j}-r_{i,j})r_{i,j+1} + (r_{i+1,j+1}-r_{i,j+1})r_{i+1,j}\}.$

In this way, the most natural finite difference discratization of the Maxwell's equations on a staggered grid system (Fig. 2) is obtained.

If the differences of the material properties spread linearly across a grid zone instead of across the interface, then there is no need to impose the boundary conditions (2) at the material interface, because the boundary condition for the tangential component of $\underline{E}$ is satisfied automatically and the other three boundary conditions are also satisfied automatically but approximately. In this way, there is no cumbersome instruction and treatment at the interface to slow down the calculation on the computer. The discretization of the terminating condition (3) is

$$E_{I,j+\frac{1}{2}} = (\mu_o/\varepsilon_o)^{\frac{1}{2}} H_{I-\frac{1}{2},j+\frac{1}{2}},$$
$$j = 0,1,2,3,\ldots\ldots,J-1.$$

To organize the above discretized (1)-(3) into a linear algebraic system, we first decompose the complex electromagnetic fields into their real and imaginary parts, i.e., $\underline{E} = \underline{E}^* + i\underline{E}^\#$ and $\underline{H} = \underline{H}^* + i\underline{H}^\#$. Then the three discretized complex scalar field equations become six real scalar field equations. Next, let the components of the unknown field vector $\underline{X}$ be arranged cyclic in "j" for each half integer incremental increasing in "i", i.e.,

$$\begin{aligned}\underline{X} = (&E^*_{\frac{1}{2},1},E^*_{\frac{1}{2},2},\ldots,E^*_{\frac{1}{2},J},/E^\#_{\frac{1}{2},1},E^\#_{\frac{1}{2},2},\ldots,E^\#_{\frac{1}{2},J},/H^*_{\frac{1}{2},\frac{1}{2}},H^*_{\frac{1}{2},1\frac{1}{2}},\ldots,H^*_{\frac{1}{2},J-\frac{1}{2}},/\\ &H^\#_{\frac{1}{2},\frac{1}{2}},H^\#_{\frac{1}{2},1\frac{1}{2}},\ldots,H^\#_{\frac{1}{2},J-\frac{1}{2}},/E^*_{1,\frac{1}{2}},E^*_{1,1\frac{1}{2}},\ldots,E^*_{1,J-\frac{1}{2}},/E^\#_{1,\frac{1}{2}},E^\#_{1,1\frac{1}{2}},\ldots,E^\#_{1,J-\frac{1}{2}},/\\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots,/E^*_{I,\frac{1}{2}},E^*_{I,1\frac{1}{2}},\ldots,E^*_{I,J-\frac{1}{2}},/E^\#_{I,\frac{1}{2}},E^\#_{I,1\frac{1}{2}},\ldots,E^\#_{I,J-\frac{1}{2}})^T,\end{aligned}$$

let the known source vector be

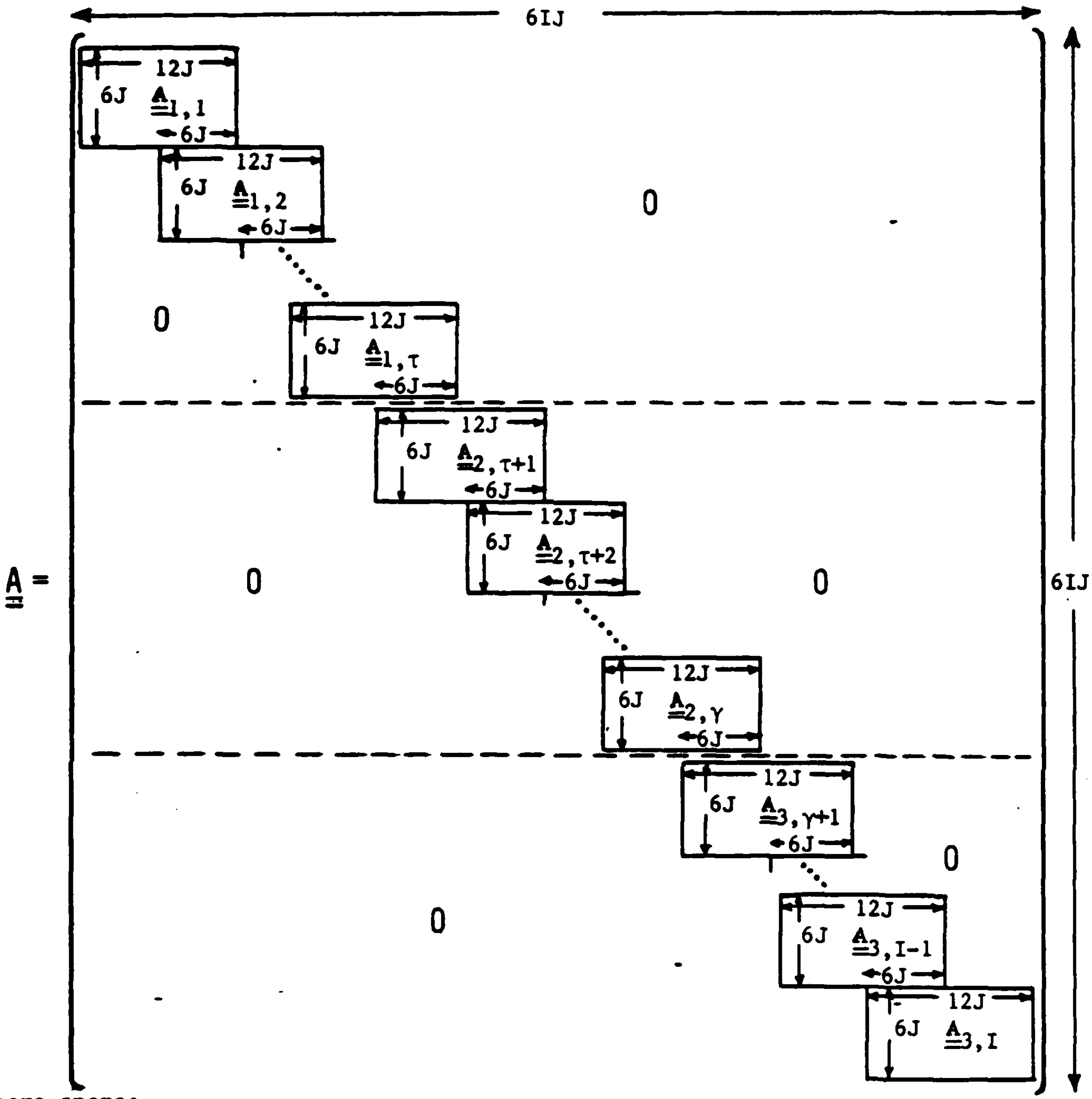
$$\underline{B} = (0,0,0,0,\ldots\ldots\ldots\ldots,b_1,b_2,\ldots\ldots,b\ )^T,$$

where the non-zero components b's depend upon the spatial distribution of the source $\underline{J}$,

and the system matrix $\underline{\underline{A}}$ is a non-symmetric band-structured sparse matrix which is shown in the following:

Let Ω_1 be defined by $i = 1,2,3,\ldots,\tau$, Ω_2 be defined by $i = \tau,\tau+1,\ldots,\gamma$, and Ω_3 be defined by $i = \gamma,\gamma+1,\ldots,I$.

Then



where sparse matrices $\underline{\underline{A}}_{1,1} = \underline{\underline{A}}_{1,2} = \cdots = \underline{\underline{A}}_{1,\tau}$ correspond to (i,j) in Ω_1, $\underline{\underline{A}}_{2,\tau+1} = \underline{\underline{A}}_{2,\tau+2} = \cdots = \underline{\underline{A}}_{2,\gamma}$ correspond to (i,j) in Ω_2, and $\underline{\underline{A}}_{3,\gamma+1} = \underline{\underline{A}}_{3,\gamma+2} = \cdots = \underline{\underline{A}}_{3,I-1} \neq \underline{\underline{A}}_{3,I}$ correspond to (i,j) in Ω_3.

Because $\underline{\underline{A}}\ \underline{X} = \underline{B}$ will be solved many times for different $\underline{B}$'s, the method of LU-decomposition will be used to solve this linear algebraic system for saving computer times. At this moment, serious programming effort has just begun.